19. Evaluate $\binom{n}{j}$.

$$\binom{7}{3} = \frac{7!}{3!\,4!} = \frac{7\cdot 6\cdot 5\cdot \cancel{4!}}{3!\cdot \cancel{4!}}$$

$$= \frac{7\cdot\cancel{6}\cdot 5}{\cancel{3\cdot 2}} = \boxed{35}$$

$$\binom{100}{98} = \frac{100!}{98!\,2!} = \frac{100\cdot 99\cdot \cancel{98!}}{\cancel{98!}\cdot 2} = 50\cdot 99$$

$$= \boxed{4950}$$

20. Expand using the Binomial Theorem.

a) $(x-1)^5$

Expand $(a+b)^5$ first:

$$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Substitute: $a = (x)$, $b = (-1)$

$$(x)^5 + 5(x)^4(-1) + 10(x)^3(-1)^2 + 10(x)^2(-1)^3 + 5(x)(-1)^4 + (-1)^5$$

$$\boxed{x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1}$$

b) $(2x+3)^5$

using the expansion above for $(a+b)^5$

Substitute: $a = (2x)$, $b = (3)$

$$(2x)^5 + 5(2x)^4(3) + 10(2x)^3(3)^2 + 10(2x)^2(3)^3 + 5(2x)(3)^4 + (3)^5$$

$$\boxed{32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243}$$

21. Use the Binomial theorem to find the indicated coefficient or term.

a) Find the coefficient of $\underline{x}^{1}$ in the expansion of $(2x-1)^3$.

$n=3$
$j=1$

$\binom{3}{1}a^1b^2$ $\quad a=2x, \; b=-1$

$\binom{3}{1}(2x)(-1)^2 = 3\cdot 2x\cdot 1 = 6x$

coefficient is $\boxed{6.}$

b) Find the third term in the expansion of $(x-3)^7$.

Third term: $n=7$, $j=2$ $\quad \binom{7}{2}a^5b^2 \Rightarrow$ $a=x$, $b=(-3)$

count by 0, 1, 2 → 1st, 2nd, 3rd

so $\binom{7}{2}(x)^5(-3)^2$

$21\cdot x^5\cdot(9) = \boxed{189x^5}$

2. Simplify $\frac{15!}{12!} = \frac{15\cdot 14\cdot 13\cdot \cancel{12!}}{\cancel{12!}} = 2730$